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Heat shield design for re-entry and launch. The use of conduction-assisted radiation on sharp-edged wings

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The conduction of heat within the material of a wing leading edge can be treated both simply and (usually) accurately by 'conducting-plate theory'. The theory leads to a concise 'reference solution' for the variation of temperature over a solid wedge-shaped nose that is heated by a boundary layer. This indicates the best choice of material, and we look at ways a given mass of the material may best be installed to reduce the nose temperature, without sacrificing the benefits that a sharp-nosed wing may have to offer.

It is shown that although these materials may be dense, a heavy mass is not required to achieve acceptable temperatures at the edge in hypersonic flight. Something perhaps between 5 and 10 kg per metre of edge (or say 3–7 lb ft^{-1}) is usually enough. However, the nose temperature depends on sweepback and surface pressure. To avoid temperatures of 1600 K or more at a highly swept *sharp* edge in hypersonic flight, it is necessary that neither the wing loading nor the surface pressure exceeds more than 2 kPa (40 lbf ft^{-2}). We cite values of less than this that relate to the design of a re-entry vehicle with a wing loading of only 680 Pa ($14\frac{1}{4}$ lbf ft^{-2}).

However, rounding the nose (with a radius usually of just a few millimetres) can provide reductions of up to perhaps 20% in the nose temperature. This allows this form of temperature control to be extended to wings of higher loading and to regions of lower sweep, including at or near the wing apex where the heating rates are most intense.

Keywords: re-entry heating; leading edge design; conduction-assisted cooling; conducting-plate theory; sharp-edge temperature; effect of rounding

1. Introduction

It is usually accepted without question that aircraft intended for hypersonic flight must have rounded leading edges. This is, of course, to avoid the high temperatures that would otherwise exist at those edges using conventional thermal protection systems. The physical principle and the technology involved are well known. Rounded leading edges spread out the peak in boundary-layer heat transfer at the nose. In protracted flight, this allows the heat to be radiated away over a larger surface area at lower temperature. A layer of insulation limits the heat transmitted to the interior structure.

However, if a high lift-to-drag ratio is sought, or if the boundary-layer transition is to be delayed, then the need for rounded edges will limit what could otherwise be

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achieved. That limit will also be present more generally, whenever an extended flight at hypersonic speeds is envisaged; for example, in seeking to extend cruise range, or the crossrange in lifting re-entry. In such a context, there is a possible remedy. The high peak heat transfer from the boundary layer over a *sharp* nose can be spread out downstream by thermal conduction within the nose material. In principle, it can then be radiated away at an acceptably low material temperature. Further, if that material is insulated downstream, the heat transmitted to the rest of the structure can likewise be limited.

This is the principle of conduction-assisted radiative cooling. The physics involved would not be disputed, and neither would the benefits that can be derived from the use of 'sharp edges' (not, let it be noted, necessarily razor sharp). Nonetheless, it would be fair to say that the idea is often dismissed as unrealistic. There are good reasons for this, in at least some contexts.

(1) The idea is only applicable if the surface pressure (and so the heat transfer) is low. In practical terms, this translates into upper limits on wing loading and dynamic pressure. This may well rule out certain applications: where, for example, a high dynamic pressure is judged essential (for effective air-breathing propulsion during launch, say).

(2) It is difficult to predict the temperature likely to be reached at a sharp edge. Frequently, reviews of the subject ignore the principle altogether, presumably for this reason. The ability to compute an answer easily has a large bearing on how acceptable any method of solving a problem may be. By comparison, working out the leading edge temperature on round-nosed wings is relatively straightforward.

(3) Suitable nose materials need to be selected from graphite and various (non-alloyed) metals with a high melting point, such as niobium (i.e. columbium), molybdenum or tungsten. The use of such metals is often regarded as implying a 'heavy' structure, but this is not so, as the relevant question concerns what effect a certain limited mass of material may have. However, the surfaces of these materials, when heated to high temperature, react with the air, although neither the rate nor the extent of the reaction is well established. Some form of surface protection may well be essential.

This last-mentioned possibility was closely studied in the 1960s and 1970s. To mention just one of the materials considered, a columbium alloy was found to accept a protective coating that allowed it to be reliably used at temperatures of up to 1800 K. It could also, to some extent, be re-used, subject to eventual problems with creep deformation. Even with the coating removed from small areas, the material was judged likely to survive several re-entries before any hole in the skin would develop, and many more before the panel would fail.

It remains an open question as to whether any such coating is effective on the almost-pure metals that we are interested in here. However, even if not, it is possible that in some applications (such as the re-entry vehicle to be cited in what follows), the leading edge protection could be routinely replaced. One advantage of metals, after all, is that they can be recycled. If the worst comes to the worst, all that is then required is that the material survives the oxidation and erosion of a single mission, whatever that might be.

In reflecting on this area of uncertainty, it will be recognized that the solution of many technological problems is driven by demand. These problems are only overcome when the benefit of doing so is clearly set out. This then is our purpose in what follows. To this end, we must address the other two problems mentioned above, which are clearly related to each other. It seems sensible to look first at methods of calculation before discussing what they reveal.

2. Methods of calculation

Conductivity usually affects such a narrow region close to the leading edge of a wing that it is adequate to treat the temperature distribution as locally two dimensional. This clearly simplifies the solution of the heat-conduction equation. However, there remain some difficulties. Firstly, the need to include the effect of surface radiation makes the equations nonlinear. Secondly, if (as we are to assume) the nose is sharp, the heat transfer has (in theory) a half-order singularity at the edge. Even if the edge is rounded, the heat transfer will vary rapidly over the (small) nose radius. Hence, the computational grid must be particularly closely spaced near the nose. The grid may also be complicated by the need to represent the internal structure of the nose and the different materials of which it may be made. Finally, the equation must be iterated with a solution of the laminar boundary-layer equations. The latter must provide a measure of the surface heat transfer appropriate to the most recently updated estimate of surface temperature. This may require real-gas effects and viscous interaction to be taken into account.

The heat-conduction equation has, itself, to be solved iteratively of course. Thus, the process can be arranged as a boundary-layer solution followed by, say, N iterations of the conduction equation. This would be repeated until convergence is achieved. At least at high Mach numbers, where heat transfer is not greatly affected by the likely range of surface temperature, N can be quite large, and convergence is relatively rapid.

Clearly, it takes some effort to set up such a computation, and each new application usually requires some reprogramming. In any general survey or project analysis, this is daunting enough to encourage one to look for a simpler approach. If nothing else, this can at least provide a close starting solution for the full calculation in chosen cases. Such an approximation, called *conducting-plate theory*, was proposed by the author over 40 years ago (Nonweiler 1952, 1956) and verified some years later by experiment (Nonweiler *et al*. 1971). Since then, its accuracy has been broadly confirmed in a large number of relevant examples by using the unsimplified approach.

(*a*) *Conducting-plate theory*

The basic assumption of this theory is that the two-dimensional domain affected by heat conduction is sufficiently thin that the temperature change across the region can be ignored compared with that along it. We can express this another way by regarding the thickness of the region as becoming infinitesimal, so that in the limit it shrinks to a plate, whence the name of the theory. The two-dimensional temperature distribution, varying along and across the domain, becomes replaced by a one-dimensional variation, in effect along the surface. However, clearly more than just shape is involved for this to be true. For instance, the leading edge of a wing may

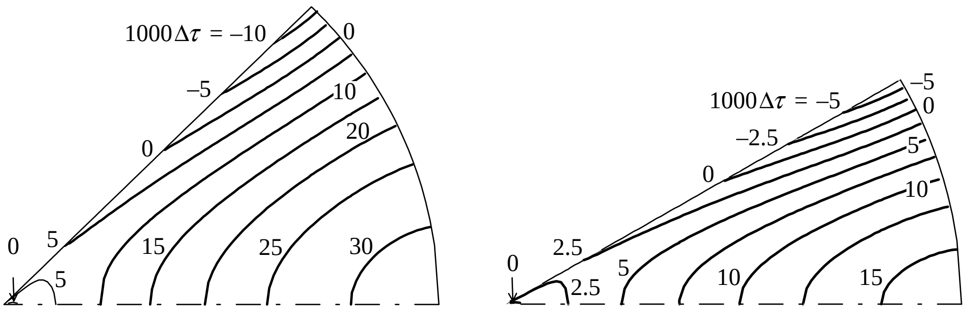

Figure 1. Difference in estimates of non-dimensional temperature: $\Delta\tau = \Delta T/\Theta = [T_2(r,\theta) - T_1(r)]/\Theta$ for symmetrical wedge sectors of 30° and 45° semi-angle and non-dimensional radial length $L/\Lambda = 1$.

be shaped as a thin wedge, but if it is filled with insulation and heated on only one side, that side will get hot while the other side will remain cool, violating the basic assumption about the temperature distribution. As with many approximations, the theory must be applied with discrimination. Yet it can also be more accurate than one might expect.

The simplest and most striking example of this is provided by a wedge sector, constructed of material of constant conductivity and insulated at its downstream end. We suppose that its top and bottom surfaces, which are cooled by surface radiation with constant emissivity, are subjected symmetrically to the same heat input, varying as $1/\sqrt{r}$, where r is the distance from the nose. Figure 1 shows the difference in the temperature between that calculated by the unsimplified two-dimensional solution (T_2), and that determined by conducting-plate theory (T_1), for just the top half of the wedge. This difference is quoted as a fraction of Θ, and, for present purposes, all that we need to know is that Θ is equal to about $T_0/1.7$, where T_0 is the value of T_1 at the edge. Thus, if Θ were as high as, say, 1000 K (1800 °R), then the edge temperature would be 1700 K (3060 °R). Even in that extreme, the approximate temperature is too small by little more than 30 K (54 °R) or less than 2% of the nose temperature. On the wedge surface, it is too small near the nose by less than a tenth of this, and too large further downstream by just 10 K (18 °R). This, let it be noted, is for a wedge of 45° semi-angle, which would hardly be described as 'thin'. For a wedge of 30° semi-angle, the temperature differences are roughly halved. The error is closely proportional to the square of the angle.

It has to be admitted that this kind of agreement is, to some extent, fortuitous. There is a good reason why (for instance) one might have included $\tan\delta$ in place of δ in the approximate treatment. Had this been done, the values of T_1 for $\delta = 45°$ would all have been 5% lower. Yet this difference is not theoretically significant, because it is of second order in δ, and, indeed, it is only numerically significant because we have chosen so large a value of δ. Had we not calculated the error, we would not, of course, know whether δ or $\tan\delta$ would provide the more accurate answer. However, we can discern that using δ instead of $\tan\delta$ provides a larger estimate of T_1, and this might be a prudent reason for preferring it.

This kind of ambiguity is inevitable, of course, in any first-order approximation. On the other hand, as we shall show, the compensating advantage is that T_1/Θ is

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independent of δ. Thus, the variation of T_1 with δ becomes embodied in the definition of the reference temperature, Θ.

Thus, not merely the working, but the presentation, of results is greatly simplified, and we have, therefore, used the theory to derive the data that follow. As in the above example, the author is confident that any error in predicting temperature should be 'small', even if the shape is not particularly 'thin'. The form of the (ordinary) differential equation in accordance with the theory will be found in the appendix, where we also suggest how it may best be solved.

3. The 'reference' result

It is our intention to show the extent to which conductivity can limit the temperature of a wing leading edge, and the ways in which the conducting material is best employed. In view of the number of variables involved, a convenient way of doing this is by quoting a basic, or what we call a *reference*, result, and expressing all others by the factor that must be applied to it. The convenience arises from the fact that this factor is never greatly different from unity.

In this sense, the reference result is also a representative or typical one. However, that remains to be shown. The basic assumptions are, perhaps, too simple and too crude for this to be immediately obvious.

The result we choose is a generalization of that already described in the previous section. We suppose that a wedge of small semi-angle δ is constructed of material of constant conductivity k, but is insulated at its downstream end at a distance $r = L$ from the nose. We assume that the total rate of heat transfer from the boundary layer to both its surfaces within a distance r of the nose is equal to $2H\sqrt{r}$, where H is a constant. We suppose further that the surface emissivities are constant and that their sum is equal to Σ. Notice that neither the heat transfer nor the emissivity need be the same on the top and bottom surfaces. Moreover, because δ is small (or, more strictly, infinitesimal), the wedge can be regarded as a triangle rather than a sector, and the radial length r does not differ significantly from the distance x along the wedge centreline, which is the measure we shall prefer to use in what follows. Then, according to conducting-plate theory, in the limit for $\delta \to 0$, the temperature T of the wedge is given by

$$T/\Theta = f(L/\Lambda, x/\Lambda), \tag{3.1}$$

where the *reference temperature* Θ and the *conduction length* Λ are defined, respectively, by

$$\Theta = \left[\frac{H^2}{\Sigma\sigma k\delta}\right]^{1/5} \quad \text{and} \quad \Lambda = \left[\frac{(k\delta)^4}{\Sigma\sigma H^3}\right]^{2/5}. \tag{3.2}$$

Here, σ is the Stefan–Boltzmann constant, $5.6697 \times 10^{-8}\ \mathrm{W\,m^{-2}\,K^{-4}}$ ($1.7121 \times 10^{-9}\ \mathrm{Btu\,h^{-1}\,ft^{-2}\,{}^{\circ}R^{-4}}$). Other than in the association of Σ with σ, and of k with δ, these relations (3.1) and (3.2) can be found from the principle of dimensional homogeneity. The form of the function $f(\dots)$ in (3.1) is determined by the solution of a (nonlinear) ordinary differential equation, given in the appendix.

There are, of course, other ways in which (3.1) might be expressed, and a number of other possible groupings of the variables that might have been taken as a 'reference'

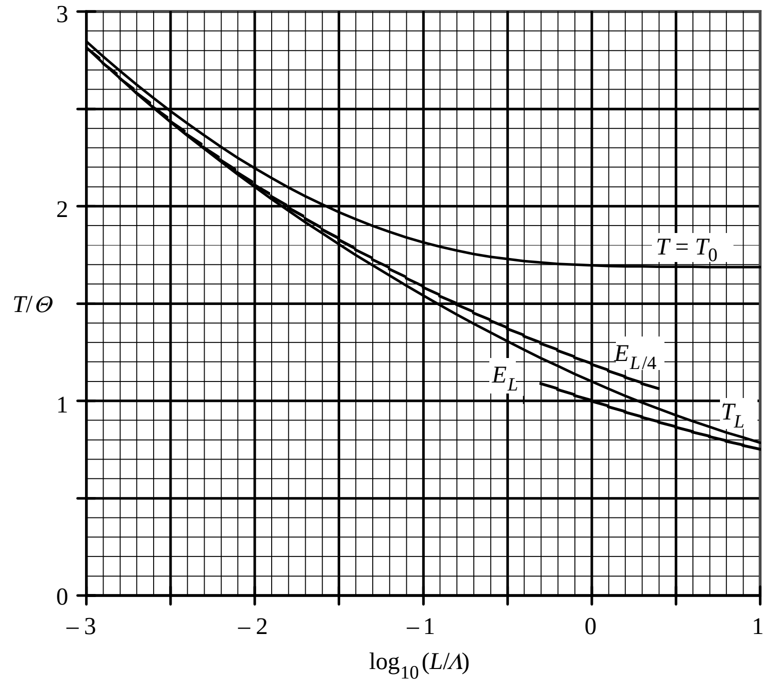

Figure 2. Variation of nose and downstream edge temperatures T_0 and T_L and of radiation equilibrium temperatures $E_{L/4}$ and E_L on a wedge as a function of its length L in the reference solution.

temperature. In particular, the radiation equilibrium temperature at a distance r from the nose is

$$E_r = [H/(\Sigma\sigma\sqrt{r})]^{1/4}. \tag{3.3}$$

It can be readily shown that

$$\Theta = E_\Lambda = (L/\Lambda)^{1/8} E_L, \tag{3.4}$$

so that, in physical terms, Θ is the radiation equilibrium temperature at a distance Λ from the nose.

For present purposes, the reason for using Θ is that, like Λ, being independent of L, the non-dimensional quotient T/Θ can be plotted, as in figures 2 and 3, to reveal directly how temperature varies with L and x. Thus, it will be clear from figure 2 that there is little reduction in the nose temperature T_0 to be achieved by increasing the length of the wedge beyond Λ. For lengths much less than about 0.01Λ, the temperature over the whole wedge is nearly constant and equal in the limit to $2^{1/4}E_L = 1.189E_L = E_{L/4}$. This limit is the constant temperature solution for 'infinite conductivity' (that is, for $k\delta \to \infty$). In this, as indeed in all solutions, the heat radiated from the wedge exactly balances that input, because we assume there to be no loss of heat from the downstream end. As the length of the wedge is increased, figure 2 also shows that the temperature T_L at the downstream end of the conducting region tends towards the radiation equilibrium temperature E_L immediately downstream. There is, consequently, a smaller temperature change across the layer of insulation we assume to be inserted at that end.

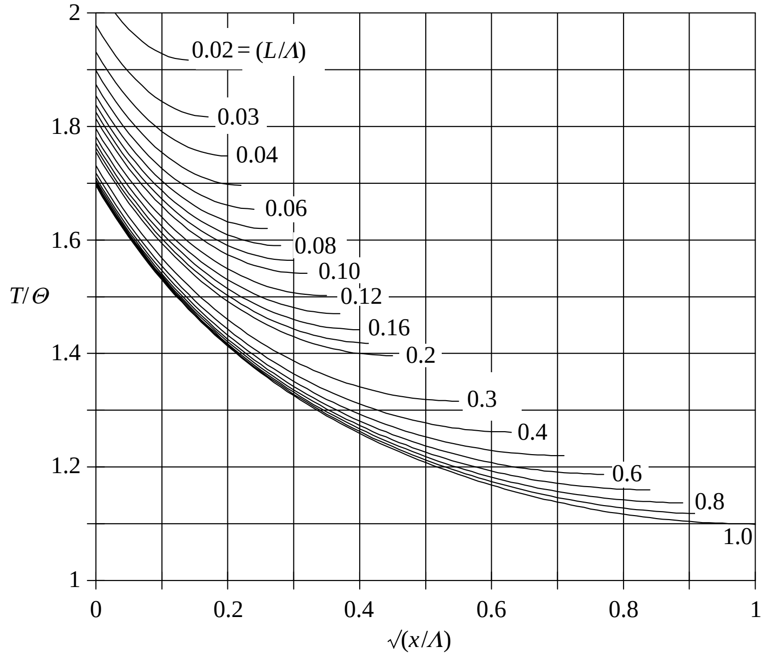

Figure 3. Variation of non-dimensional temperature T/Θ with distance x along a wedge subjected to a heat input proportional to $1/\sqrt{x}$ for different values of its non-dimensional length L/Λ.

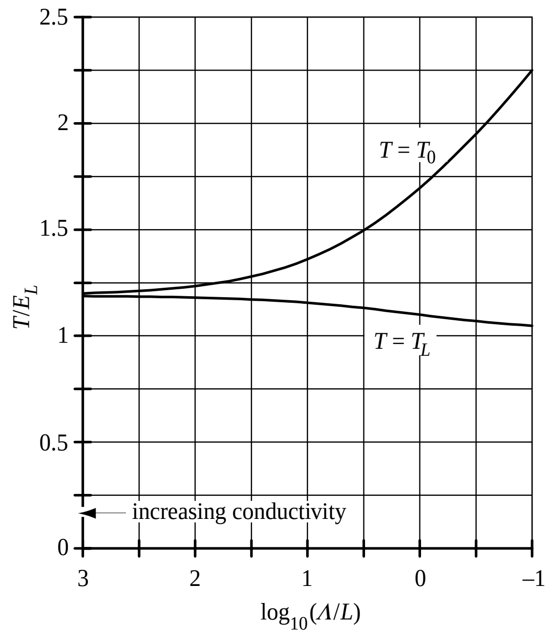

Figure 4. Another presentation of the results for the reference solution to show the effects of material conductivity.

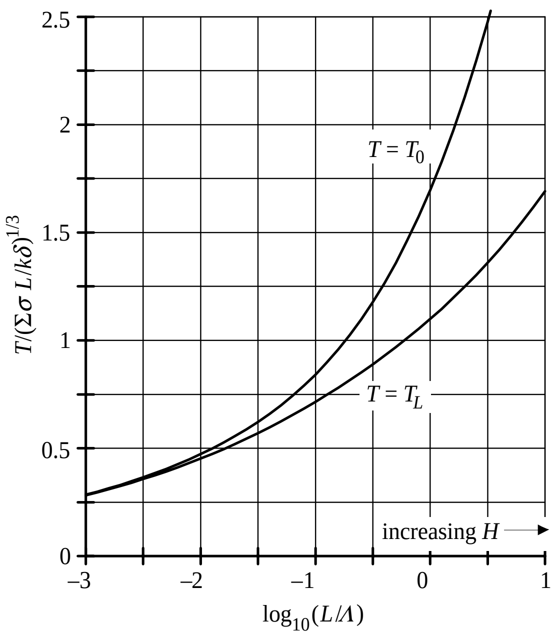

Figure 5. Another presentation of the results for the reference solution to show the effects of heat input intensity.

On the other hand, if (for instance) we plot T/E_L against Λ/L, as in figure 4, then, because E_L is independent of conductivity, this reveals more clearly how the temperature is affected by $k\delta$, since, from (3.2), the latter is proportional to $(\Lambda/L)^{5/8}$. Yet another approach is to plot

$$T\Big/\left[\frac{k\delta}{\Sigma\sigma L}\right]^{1/3} = \left(\frac{T}{\Theta}\right)\left(\frac{L}{\Lambda}\right)^{1/3}$$

against L/Λ, as in figure 5. This exhibits best how T increases with H, which is proportional to $(L/\Lambda)^{5/6}$. Informative as these representations may be, they all of course convey precisely the same result. In what follows, we shall find it convenient for the most part to use T/Θ as a measure of non-dimensional temperature.

We question first of all whether the reference solution adequately represents the effect of aerodynamic heating at hypersonic speeds. By considering (in § 3 *b*) what conducting materials may best be used, we are led to some idea of the heating intensities that they can sustain. Reference to laminar boundary-layer rates of heat transfer (in § 3 *c*) then suggests the relatively low wing loading and the high sweepback that seem, at least at first sight, to be implied by these intensities.

(*a*) *Representation of boundary-layer heat input*

If it is intended to represent the effect of heat input from a laminar boundary layer, perhaps the least realistic assumption of the reference solution is that the total heat input to the wedge is proportional to $\sqrt{x}$. One's first doubt would arise about the reality of the half-order singularity at the nose. This could only exist (in theory) if the surface were flat and at constant pressure, whereas in practice the nose would, at least to some extent, be rounded. However, this fact leads to complications, and it will be more convenient to come back to unravel these later. For the time being, we shall accept that the wedge surface can be treated as if it were in fact at constant pressure.

Even so, if the heat input $H\sqrt{x}$ is supposed to be derived from a boundary layer, then H will not be a constant (as has been assumed). It will increase downstream, where the surface temperature of the wedge is less. Similarly, H at $x = 0$ will vary with nose temperature T_0. The latter dependence has by far the greater effect on T_0 itself, and, fortunately, it can be taken into account in the reference solution. It simply means that placing $T = T_0$ in (3.1) provides an implicit, rather than an explicit, expression for T_0, and consequently for H. The fact that conductivity also varies (usually only gradually) with temperature, causes a similar difficulty. Most of the error from this source can likewise be avoided by taking k as dependent just on T_0.

Near to the leading edge, the value of H decreases with T_0 closely in proportion to $(1 - h_0/h_{\text{rec}})/(1 + h_0/h_{\text{c}})$. Here, h_0 is the air enthalpy at the surface at temperature T_0 at the nose, h_{rec} is that at the recovery temperature, and h_{c} is a constant enthalpy of *ca.* 55 km^2 s^{-2} (6×10^8 ft^2 s^{-2} or 24 000 Btu lb^{-1}). Clearly, for any realistic surface temperature, the variation of H is quite small at hypersonic speeds.

(*b*) *The selection of conducting material*

As will be shown, the triangular slab assumed in the reference solution does not make the best use of the conducting material. Nonetheless, this solution is realistic enough to guide us to the best choice of conducting materials. It can also provide at least an underestimate of the level of heat input H that they must sustain. For this purpose, we need to make a few extra assumptions.

(i) We shall take the sum of the surface emissivities Σ as 1.7, and δ to be 20°. This latter angle is possibly large, but we have in mind the leading edge of a highly swept wing, and the semi-wedge angle in the streamwise direction would be smaller, perhaps just 5°.

(ii) We shall suppose that the mass m of the conducting material per unit length along the wing leading edge is fixed. The length of the wedge of material is then $L = \sqrt{(m/\rho\delta)}$, where its density is $\rho = 1000 s_{\text{g}}$ kg m^{-3}, and s_{g} is its specific gravity. An acceptable value of m will depend on the size of the aircraft. In the recent design (Nonweiler, this issue, SLEEC22) of a small re-entry vehicle of 4.4 m ($14\frac{1}{2}$ ft) span, m was limited to 10 kg m^{-1} (6.7 lb ft^{-1}), but values of up to perhaps 30 kg m^{-1} (20 lb ft^{-1}) might be acceptable on larger vehicles, if this proved a benefit.

(iii) We shall take both T_0 and H to be given as parameters, and plot the values of s_{g} and the conductivity k that are thereby implied, as in figure 6*a*–*d*. For this purpose, we need yet another representation of the reference solution, in which (in effect) we plot

$$\left(\frac{T_0}{\Theta}\right)^5 = \frac{k\Sigma\sigma T_0^5\delta}{H^2} \quad \text{against} \quad \left(\frac{T_0}{E_L}\right)^{-16} = \left(\frac{\rho\delta}{m}\right)\left[\frac{H}{\Sigma\sigma T_0^4}\right]^4.$$

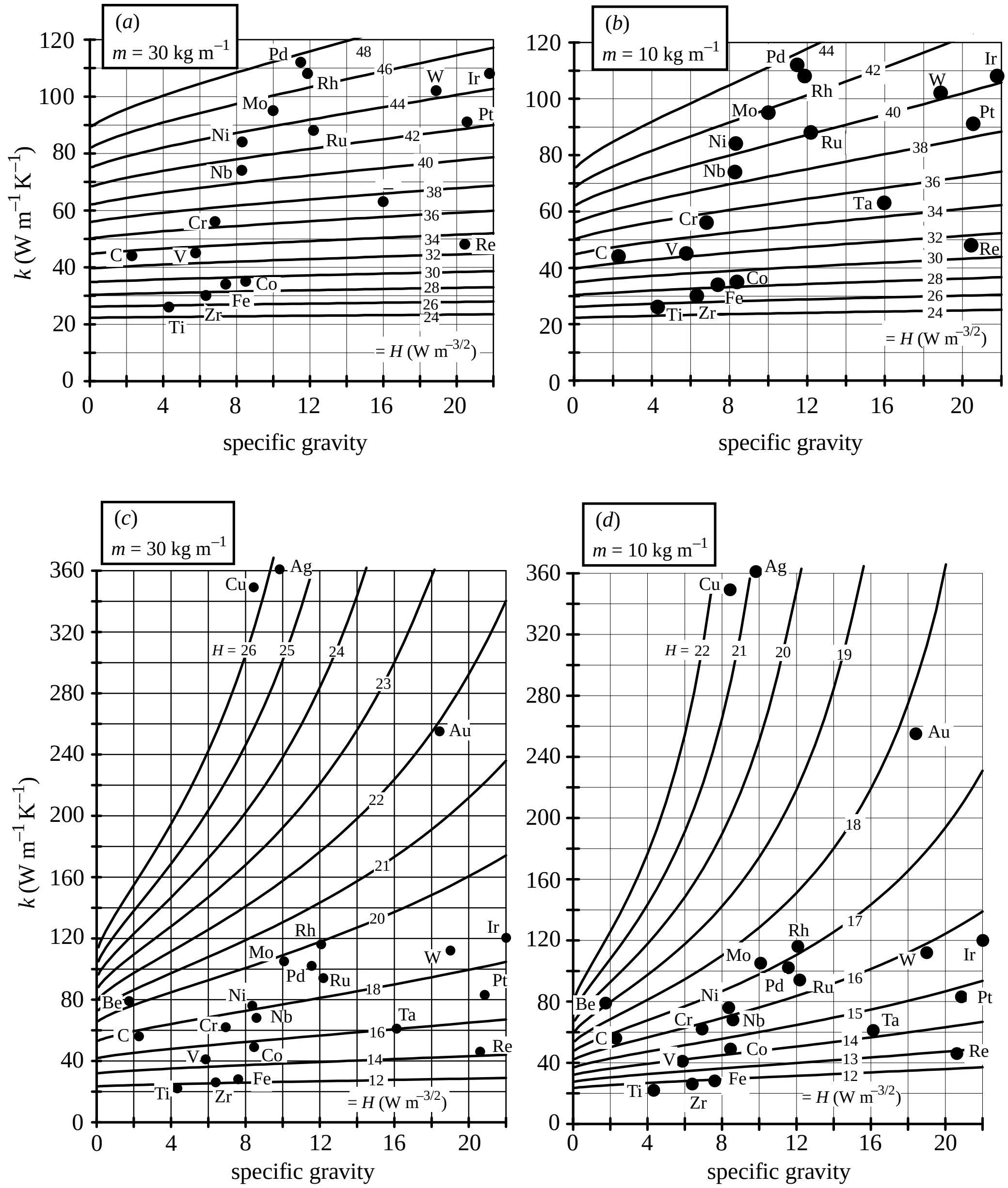

Figure 6. Effect of material conductivity, density and mass per unit length on the heat transfer intensity H that corresponds in the reference solution in (*a*) and (*b*) to a nose temperature $T_0 = 1600$ K, and in (*c*) and (*d*) to a nose temperature $T_0 = 1200$ K. (The symbol C refers to carbon in the form of graphite.)

Figure 6 also shows the values of k and s_g (at the temperature T_0) for most of the elements that have melting points higher than T_0. Many of these materials will be best known for their use in alloys. However, in general, alloys have much lower conductivities than does each of their ingredients, although one form of chromium steel has physical properties like those of iron.

Looking first at figure 6*a*, we see that with $m = 30\,\mathrm{kg\,m^{-1}}$ ($20\,\mathrm{lb\,ft^{-1}}$) and $T_0 = 1600\,\mathrm{K}$ ($2880\,^\circ\mathrm{R}$) the 'best' materials, in the sense that they sustain the highest heating intensity H, are palladium and rhodium. Unfortunately, along with ruthenium, iridium and platinum that also show up well on this basis, they would be too rare and costly to be used. The choice, therefore, turns to molybdenum, tungsten, nickel and niobium. Nickel has the lowest melting point of these and is probably only usable (if at all) at a somewhat lower temperature than 1600 K.

It is clear from figure 6*b* that the reduction of the mass of the conducting material by a third to $m = 10\,\mathrm{kg\,m^{-1}}$ ($6.7\,\mathrm{lb\,ft^{-1}}$) has no effect on the 'ranking' of materials, and little effect on the heat intensity H that can be sustained. Indeed, H is reduced by only *ca.* 7% for the materials of highest conductivity, and less still for those with smaller k. If H remained fixed, and T_0 were varied, then this 7% change in H would translate to a mere 3% reduction in nose temperature. A threefold increase in the mass of the conducting material would seem an extravagant penalty to pay for so small an effect. The converse of this observation is no less important. Reducing the mass by a third (to, say, $3\,\mathrm{kg\,m^{-1}}$) would imply only a similarly small increase in T_0 (for fixed H). However, one way or the other, the significance of such a measure can only really be judged by reference to its context.

This variation (of H with m) arises because, in all of the conditions shown in figure 6*a*, *b*, the values of L/Λ are in the range approaching unity, where, as shown in figure 2, T_0/Θ becomes almost independent of length. Because T_0 is fixed, Θ is nearly constant and H becomes roughly proportional to $\sqrt{k}$, but insensitive to s_g (and so to L and m).†

Turning next to figure 6*d*, for which m remains at $m = 10\,\mathrm{kg\,m^{-1}}$ ($6.7\,\mathrm{lb\,ft^{-1}}$), but T_0 is now reduced to 1200 K ($2160\,^\circ\mathrm{R}$), we see that copper, beryllium and (to a lesser extent) graphite enter into contention as preferred materials (gold and silver being obviously out of the question). Moreover, the relative importance of density at the higher values of H and k, as judged from the steepness of the curves, suggests that the extent of the surface is, here, more important, because L is much smaller than Λ. Indeed, as we see from figure 6*c*, an increase of m to $30\,\mathrm{kg\,m^{-1}}$ is now more effective, because it allows copper to sustain a heating intensity 22% higher without an increase in nose temperature.‡

To sum up, while it is not suggested that a nose temperature of 1600 K ($2880\,^\circ\mathrm{R}$) is the highest that might be tolerated, we can, for present purposes, regard a heat intensity H of *ca.* $40\,\mathrm{kW\,m^{-3/2}}$ ($23\,000\,\mathrm{Btu\,h^{-1}\,ft^{-3/2}}$ or $5000\,\mathrm{lbf\,s^{-1}\,ft^{-1/2}}$) as representing a 'high' heat input. We regard a figure of half this as a 'moderate' value for which copper might be the obvious conducting material to use. In any application, it would of course be the peak heating intensity that would determine the choice of material.

† As indeed is clear from the figures. For instance, figure 6*b* indicates that niobium can sustain a heat input of $H = 39\ \mathrm{W\,m^{-3/2}}$ and with $k = 74\ \mathrm{W\,m^{-1}\,K^{-1}}$, equations (3.2) imply values of $\Lambda = 36$ cm (14 in) and $\Theta = 906$ K ($1630\,^\circ$R), so that $T_0/\Theta = 1.77$. However, since $s_\mathrm{g} = 8.3$, the value of L that corresponds to a mass of $10\ \mathrm{kg\,m^{-1}}$ ($6.7\ \mathrm{lb\,ft^{-1}}$) is 59 mm (2.3 in) so that $L/\Lambda \approx 0.16$, which is shown in figure 1 to be consistent with the value of T_0/Θ.

‡ For this increased H (of $26.3\ \mathrm{kW\,m^{-3/2}}$), the values of Θ and Λ are 568 K ($1020\,^\circ$R) and 7 m (23 ft), respectively. As the length of the copper wedge for a mass of $m = 30\ \mathrm{kg\,m^{-1}}$ is 10 cm (4 in), the corresponding figures for T_0/Θ and L/Λ are 2.11 and 0.014, respectively.

(c) Interpreting the value of H

Perhaps the relation (3.3) for E_r provides the easiest way of interpreting the value of H. What we have termed 'high' and 'moderate' heat intensities of 40 and 20 kW m$^{-3/2}$ correspond to radiation equilibrium temperatures of 800 K and 675 K at a distance of 1 m perpendicularly from the edge of the swept wing (or 1700 °R and 1400 °R at 1 ft from the edge). This assumes that $\Sigma = 1.7$, which is the figure used in deriving the results of figure 6.

With the leading edge being swept back (by an angle ϕ, say), it would be more usual to base the radiation equilibrium temperature E on the distance downstream from the edge in the *streamwise* direction. This relates E to a point nearer the edge and increases it by a factor of roughly $\sec^{1/8}\phi$. For instance, with $\phi = 75°$, the radiation equilibrium values E for 'high' and 'moderate' heat intensities would then be roughly 950 K and 850 K at 1 m from the edge in the streamwise direction (or 2000 °R and 1800 °R at 1 ft streamwise from the edge).

Likewise, it would be more usual to refer the heat intensity to a value (H_s say) based on the streamwise distance s from the edge. Then the total heat input to the surface over that distance s is $2H_s\sqrt{s}$. Since the distance normal to the edge is $x \cong s\cos\phi$, it follows that

$$H_s = H\sqrt{\sec\phi}. \tag{3.5}$$

Thus, the values of H_s would be almost double those for H if ϕ were equal to 75°.

A survey (Nonweiler 1990) of the results for the rate of heat transfer Q from a laminar boundary layer over a (cold) surface at constant pressure p_e, yields the approximation

$$Q\sqrt{s} = (V_\infty/V_c)^{2.16}\sqrt{Cp_eU_e/V_\infty}, \tag{3.6}$$

where U_e is the flow velocity over the surface, V_∞ is the free-stream speed, V_c is the sea-level circling velocity of 7.87 km s^{-1} (25 800 ft s^{-1}), and $C = 77$ MW s^{-1} (57×10^6 ft lbf s^{-2}). Evidently,

$$H_s = (Q_t + Q_b)\sqrt{s}, \tag{3.7}$$

where Q_t and Q_b are the values of Q for the top and bottom surfaces, respectively. Assuming these were the same, then at a speed of $V = 3$ km s^{-1} (or 10^4 ft s^{-1}), a value of H_s of 80 kW m$^{-3/2}$ would correspond to a surface pressure of somewhat more than 1.4 kPa (29 lbf ft^{-2}).

Broadly speaking, as the speed increases, the surface pressure is likely to drop in rough proportion to $1 - (V_\infty/V_c)^2$. Equation (3.5) then shows that Q is a maximum at $V_\infty/V_c = 0.83$. In this extreme condition, it is likely that the heat input to the top surface would be relatively small. Thus, a value of $H_s = 80$ kW m$^{-3/2}$ would correspond to a value of $p_b/[1 - (V_\infty/V_c)^2]$ in excess of 1.9 kPa (39 lbf ft^{-2}), where p_b is the bottom surface pressure. This, in turn, would imply a wing loading of about this same magnitude.

As a rough guide, provided the wing leading edge is highly swept, and neither the wing loading nor the surface pressure exceeds *ca.* 2 kPa (40 lbf ft^{-2}), we see that the heating intensity is likely to be 'moderate' at a Mach number of about 7, and 'high' for all Mach numbers above 10.

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Table 1. *Edge materials and the reference solution for* T_0

material	s_g	k (W m^{-1} K^{-1})	L (mm)	Θ (K)	Λ (cm)	L/Λ	T_0/Θ	T_0 (K)
graphite	2.3	43	109	879	24	0.46	1.713	1505
niobium	8.34	71	58	795	53	0.109	1.805	1430
tungsten	18.95	108	38	731	103	0.037	1.944	1420
copper	8.46	339	57	582	643	0.0089	2.222	1290

We can carry this a little further by noting that when the heat intensity is high, T_0 is almost proportional to the reference temperature Θ. Equation (3.2) then shows that the nose temperature varies as $H^{2/5}$, so, in the single-sided heating condition at high Mach number, equations (3.5)–(3.7) show that T_0 varies as $(p_b \cos\phi)^{1/5}$. A doubling of wing loading to 4 kPa (80 lbf ft^{-2}), or a reduction of sweepback from 75° to 60°, can increase the nose temperature from what we have called a high value of 1600 K (2880 °R) by 15%, to 1840 K (3000 °R). This assumes that the semi-angle δ perpendicular to the edge remains unaltered. If edge angle in the streamwise direction (δ_s, say) were to stay the same, then, since $\delta \approx \delta_s \sec\phi$, Θ and, consequently, T_0, vary approximately as $(p_b \cos^2\phi)^{1/5}$, and the effect of sweepback is even more pronounced. The result also assumes that we cannot improve on the results of the reference solution, but (as we shall now endeavour to show) we can indeed do so.

4. The design of the conducting material

While we shall describe the design of the conducting leading edge in general terms, it is useful nonetheless to refer the general results to a practical example. For this purpose we shall use the designed re-entry vehicle SLEEC22 (Nonweiler, this issue) to provide relevant figures. It has the following features.

(i) Its wing loading is only 680 Pa ($14\frac{1}{4}$ lbf ft^{-2}).

(ii) Its wing leading edges are swept back at an angle of 72°.

(iii) The semi-angle of the leading edge is 8.7° in a streamwise plane, and $\delta = 20.6°$ (0.36 rad) in the plane normal to the edges.

(iv) The descent of the vehicle is planned so that, at all Mach numbers of 10 and above, the top-surface heat transfer should be negligible, and the heating rate, $Q_b\sqrt{s}$, to its bottom surface should not exceed 50 kW m$^{-3/2}$ (28 700 Btu h^{-1} ft$^{-3/2}$ or 6200 lbf s^{-1} ft$^{-1/2}$). This corresponds to a radiation equilibrium temperature on the bottom surface (derived from (3.3) by replacing Σ with $\varepsilon_b = 0.85$) of 1010 K at a streamwise distance of 1 m from the leading edge (2100 °R at 1 ft streamwise from the edge). From (3.5) and (3.7), it follows that the value of H is not more than 28 kW m$^{-3/2}$ at Mach numbers of 10 and above, and becomes progressively smaller as the Mach number decreases below 10.

Four possible materials were considered for the conduction-assisted cooling of its wing leading edges: each restricted to a mass of 10 kg m^{-1} (6.7 lb ft^{-1}) along the

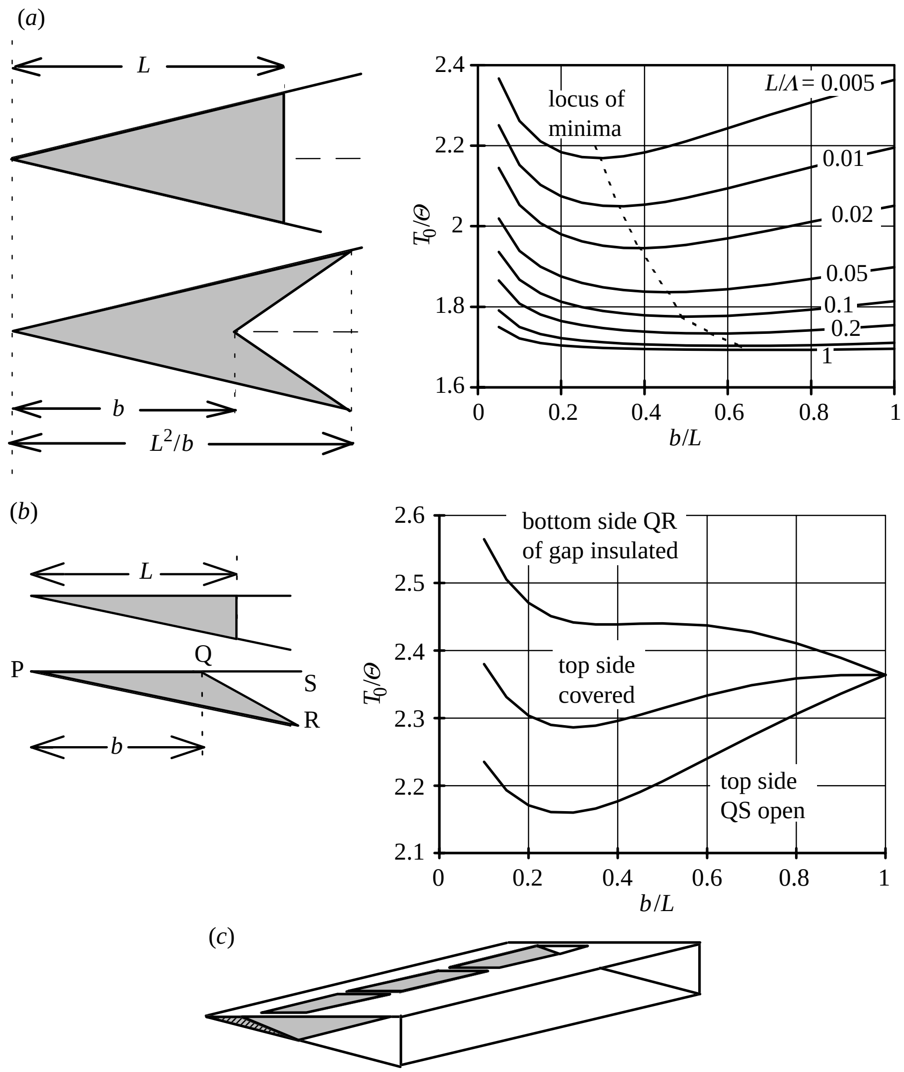

Figure 7. Effect on T_0/Θ of a linear downstream taper of the conducting material, leaving its cross-sectional area (i.e. mass) unchanged. (*a*) Top- and bottom-surface heat transfer and emissivity assumed equal. (*b*) Heat transfer only to the bottom surface and $L/\Lambda = 0.005$. (*c*) Top side open or transparent.

edge. These and their relevant characteristics are given in table 1, assuming (as in the reference solution) that the sum of the surface emissivities is $\Sigma = 1.7$.

There does not seem much to choose between the use of tungsten and niobium as a means of limiting the nose temperature, but the temperature seems too high for the use of copper (which melts at 1357 K).

(*a*) *The use of taper*

The material nearest the nose is more effective in controlling nose temperature than that downstream. As a consequence, the triangular cross-section of the reference solution is not an efficient use of a fixed mass of the material, since it continually increases in thickness downstream. Rather, it is better to taper the material off as the distance from the nose increases. This is illustrated for a symmetric heating (and so also temperature) distribution in figure 7*a*. The resulting elongation of the conducting region will be seen to have little effect if the length L of the triangular cross-section is comparable with Λ. However, as the value of L/Λ decreases, the possible gain from tapering becomes more marked.

Note that there is no heat transfer across the internal gap between the two tapered portions in this symmetric condition. Thus, the internal surfaces, or the entire gap, could be insulated, without affecting the estimate of nose temperature. However, when the heat transfer is assumed to be restricted to the bottom surface only, then, as illustrated in figure 7*b*, taper is not necessarily an advantage. If the interior of the tapered section is insulated along its interior surface (QR in the diagram), or within the entire gap between top and bottom surfaces QRS, then the external radiation is halved and taper serves only to increase the nose temperature, as shown. On the other hand, if the top surface QS is open, or transparent, so that the wing leading edge might resemble the sketch in figure 7*c*, then two-sided radiation is available (as assumed in the reference solution), and the reduction is precisely the same as that shown for symmetric heating in figure 7*a*. There is an intermediate condition where the top surface QS is solid, and if its surface emissivity is 0.85 (like all other surfaces), then, considering the radiation balance across the gap QRS, we find that the net emissivity of the surface QR is reduced by a factor of 2.15 (to 0.395). Thus, the sum of the emissivities on the two sides of the tapered portion is reduced, in effect, from the value of 1.7 (assumed in evaluating Θ and Λ) to 1.245, and, as figure 7*b* shows, the advantage of taper is largely lost.

For the re-entry vehicle described at the start of this section, the heat input at high Mach number is single sided. Provided the top surface is open or transparent, then, from table 1, we find that the introduction of straight taper reduces the nose temperatures of niobium, tungsten and copper by a maximum of 2%, 3.8% and 6.9%, respectively, consistent with the decreasing values of L/Λ for each of these materials (as given in table 1). The corresponding values of T_0 are 1405 K (2530 °R) for niobium, 1365 K (2460 °R) for tungsten and 1205 K (2170 °R) for copper.

Measured by the effect on T_0, there are more efficient forms of taper than the linear taper shown in figure 7. The optimum shape is not known, but it seems likely to entail a more gradual reduction in material thickness downstream, merging as it were into a thin ‘skin’, with a consequent increase in the total length.

(*b*) *The use of a less dense material*

The rate of conduction of heat downstream depends on the product of the material conductivity and its local thickness. Taper reduces the downstream thickness, but much the same effect can be obtained by using a material of lower density downstream. There would, of course, only be advantage in this if it does not have too low a conductivity.

For instance, figure 8 illustrates the effect of using a second material of 0.6 times the conductivity but 0.3 times the density (like graphite behind a niobium tip). Provided thermal contact is maintained between the top and bottom surfaces over the entire length, the reduction is the same irrespective of whether the heating is symmetric. In

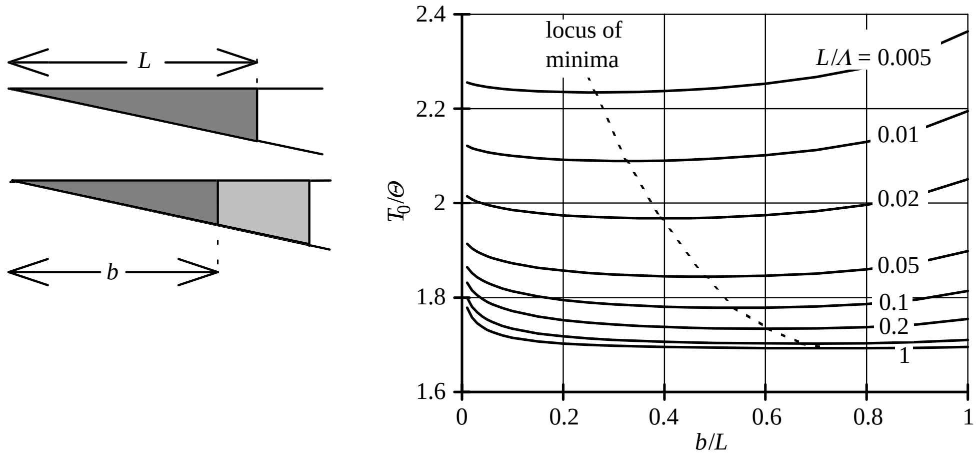

Figure 8. Effect on T_0/Θ of the downstream addition of a less-dense material, leaving the total mass unchanged. (The density ratio is 0.3 and the conductivity ratio is 0.6.)

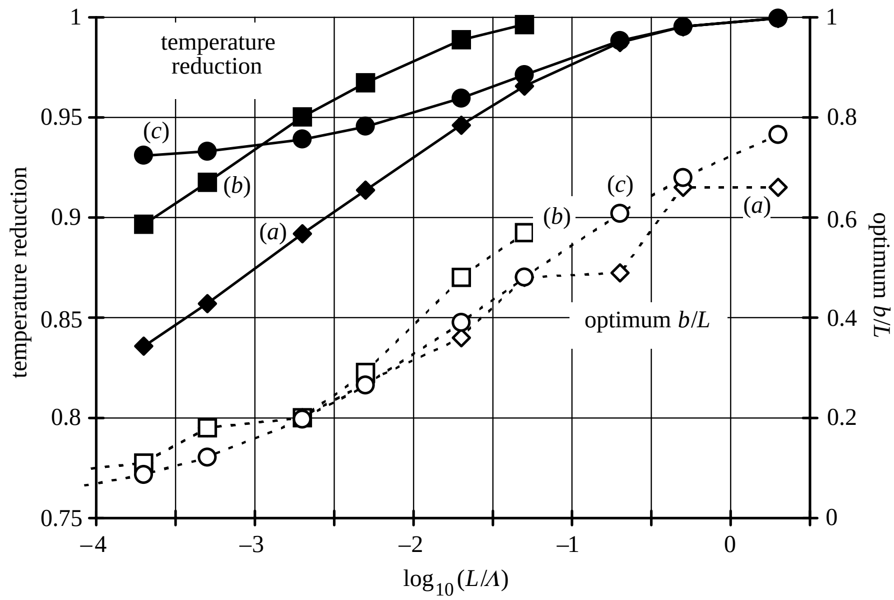

Figure 9. Optimum reduction of T_0/Θ leaving the total mass unchanged, due to (*a*) a linear downstream taper of the conducting material, with either equal top and bottom heat transfer, or with open top surface and heat transfer only to the bottom surface; (*b*) a linear downstream taper of the conducting material, with closed top surface and heat transfer only to the bottom surface; (*c*) the downstream addition of a less-dense conducting material. (The density ratio is 0.3, the conductivity ratio is 0.6, heat transfer to either or both surfaces.)

the example of the re-entry vehicle already cited, the maximum reduction in T_0 for graphite with a niobium tip is 2%, almost the same as that which can be achieved by linear taper. However, for graphite backing a tungsten tip, the maximum reduction in this example is 4.4%, rather more than for taper, with T_0 decreased from 1420 K to 1360 K (2445 °R). Copper is best backed with beryllium and this gives a maximum

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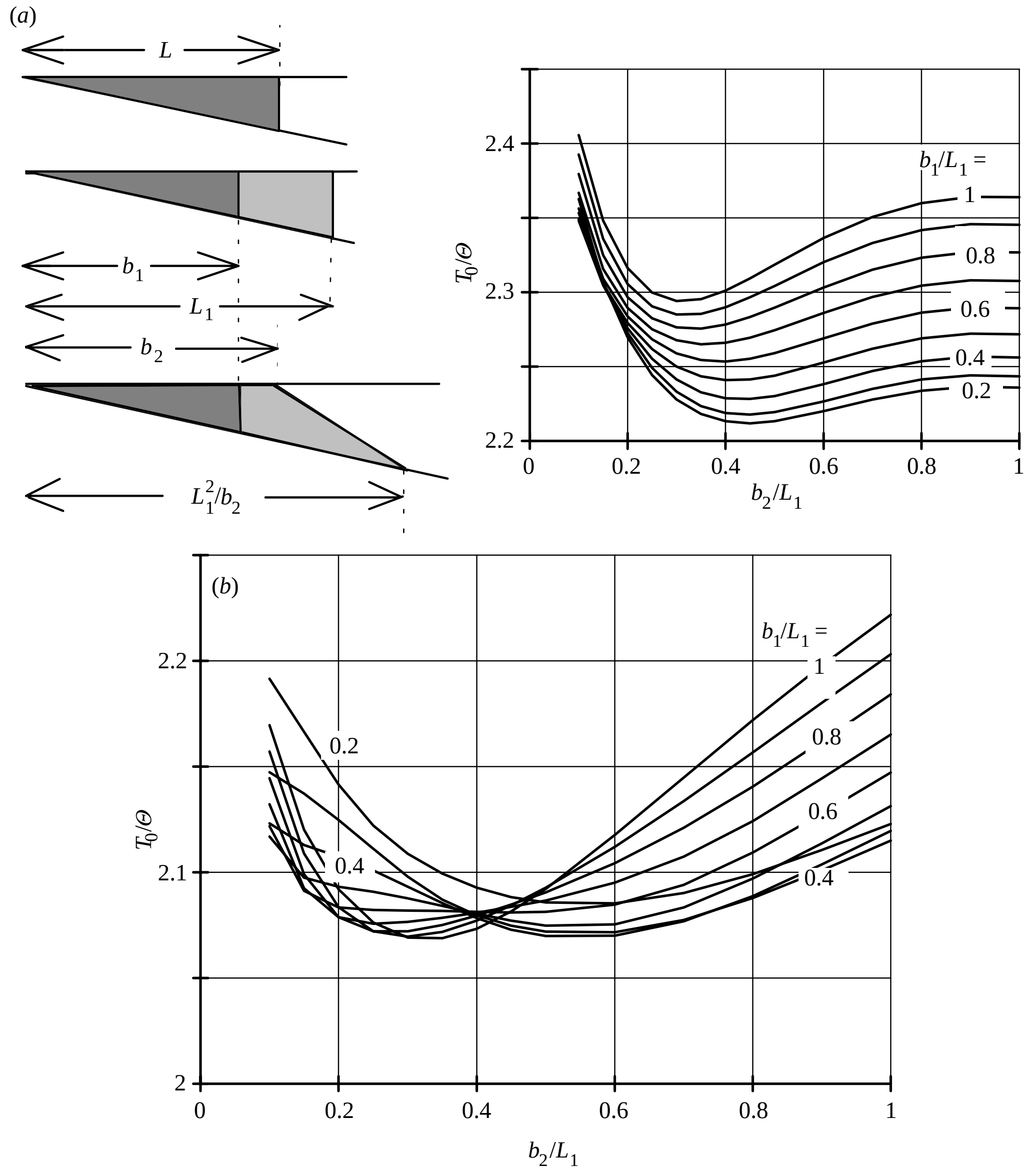

Figure 10. Effect on T_0/Θ of a linear downstream taper of the conducting material as well as the downstream addition of a less-dense material, leaving the total mass unchanged. (*a*) Closed top surface and heat transfer only to the bottom surface; $L/\Lambda = 0.005$, density ratio is 0.3, conductivity ratio is 0.6. (*b*) Either equal top and bottom heat transfer, or open top surface and heat transfer only to the bottom surface; $L/\Lambda = 0.009$, density ratio is 0.21, conductivity ratio is 0.23.

reduction in T_0 of almost 5% from 1290 K to 1230 K (2215 °R), which is less than can be achieved by taper at the small L/Λ involved. Clearly, the effect must depend on the materials used. However, as in the example of figure 9, whatever the materials, there is a limit to the reduction that can be achieved as L/Λ is reduced.

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It is generally beneficial to combine the use of a low density material with linear, or any other shape of, taper. This is illustrated in figure 10*a* for a condition of one-sided heat transfer. It will be seen that there is an interaction between the two measures, so that the resultant drop in T_0 is not simply the sum of the component reductions. In the example of the re-entry vehicle SLEEC22, if we assume, as before, that the top surface is open or transparent, as in figure 7*c*, then the maximum reduction in T_0 from this combination is increased still further to 2.8% for niobium and 5.4% for tungsten. The corresponding values of T_0 are reduced to 1395 K (2510 °R) and 1345 K (2420 °R). However, for copper, as indicated in figure 10*b*, it happens that the maximum reduction due to taper with or without the addition of a beryllium tailpiece is about the same, and we cannot better the value of 1205 K (2170 °R).

These are, at best, quite modest reductions, although certainly worthwhile. They back up the case for regarding the reference solution itself as being widely representative. However, the largest effect of all is achieved by rounding the leading edge, and this we look at in the next section.

5. The effect of rounding the nose

Clearly, a nose shape that is rounded, or otherwise blunted, must be better able than one that is sharp to conduct a given amount of heat downstream, away from this 'hot-spot'. Moreover, it is well known that the effect of rounding on the boundary layer is to spread out the peak in the rate of heating at the nose. On the other hand, this may also serve to reduce the temperature gradient that drives heat conduction, so that it cannot be assumed that rounding is entirely beneficial.

Almost all of the results that we shall quote in what follows relate to a (two-dimensional) wedge with a *circularly rounded* nose. This choice of geometry is merely a matter of convenience, as it is not clear what an optimum shape might be. The application of conducting-plate theory in this context is described in the appendix. For it to retain good accuracy, experience suggests that the nose radius should not exceed approximately one-tenth of the length of the material. However, there is no rigorous justification of this limit.

This is just one of several simplifications that seem unavoidable if we are to avoid a morass of detail. Over a 'sharp-edged' wedge, we have been willing to accept that the variation of heat input is known and that merely a constant of proportion (H) distinguishes one condition from another. But the difficulty in discussing the effect of rounding the nose is that the variation of heat transfer over the surface is dependent on wedge angle, sweepback, Mach number, and, possibly, Reynolds number as well. All that we attempt to offer here is some broad indication of the magnitude of the effect.

(*a*) *Boundary-layer heat transfer*

The variation of the rate of aerodynamic heating over a circularly rounded nose of the wedge is illustrated in figure 11. The calculations assume the hypersonic approximation for the flow of a perfect gas.† It will be seen that over the curved portion of

† The air is treated as a perfect gas with $\gamma = 1.4$. The surface is supposed to be cold, with its pressure given by the Newton formula. The boundary-layer flow is assumed laminar, with the air viscosity varying as $T^{0.65}$, and its Prandtl number equal to 0.7.

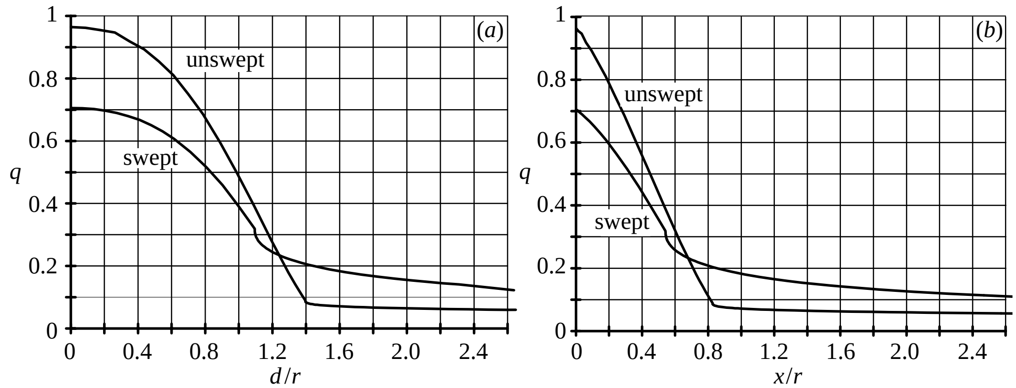

Figure 11. Variation of heat transfer coefficient q over a round-nosed wedge of radius r and streamwise semi-angle 10°, with its edge unswept, or swept back 70°. Here $q = Q\sqrt{[r/(p_0\mu_{\mathrm{tot}}H_{\mathrm{tot}}^{1.5})]}$, where Q is the local rate of heat transfer, subscript 0 refers to stagnation conditions at the edge, and μ_{tot} is the air viscosity at the temperature corresponding to the total enthalpy, H_{tot}. (*a*) q versus distance d around the nose in a plane normal to the edge. (*b*) q versus axial distance x from nose in a plane normal to the edge.

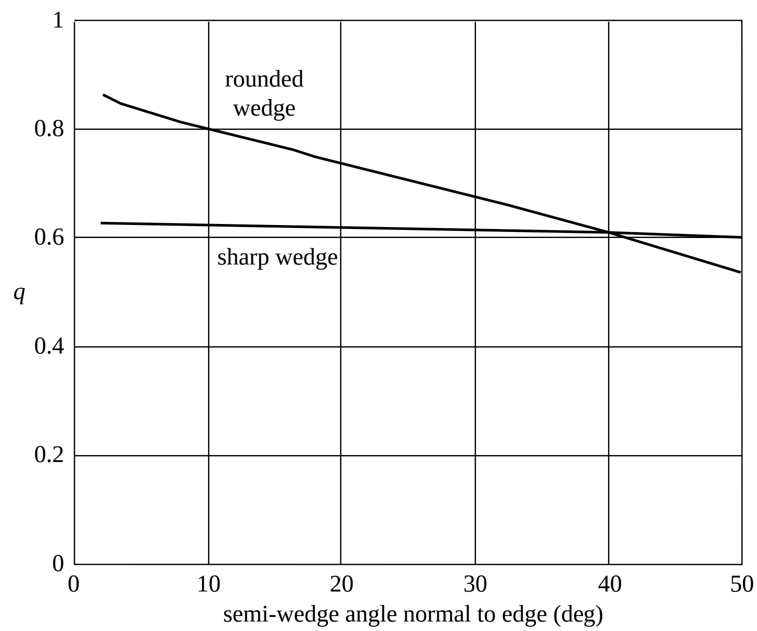

Figure 12. Asymptotic rates of heat transfer far downstream on a wedge with a rounded or sharp leading edge. Here $q = Q\sqrt{[s/(p_{\mathrm{e}}\mu_{\mathrm{tot}}H_{\mathrm{tot}}^{1.5})]}$, where s is the distance along the wedge surface in the *streamwise* direction, and p_{e} is the surface pressure.

the surface, the heating rate varies more or less linearly with axial distance x, and continues to fall, though less rapidly, along the flat portion of the wedge downstream of the shoulder. The wedge angle determines where that break occurs, but does not influence the heating upstream, over the curved surface.

Figure 12 shows that far downstream, where the effect of the nose curvature has

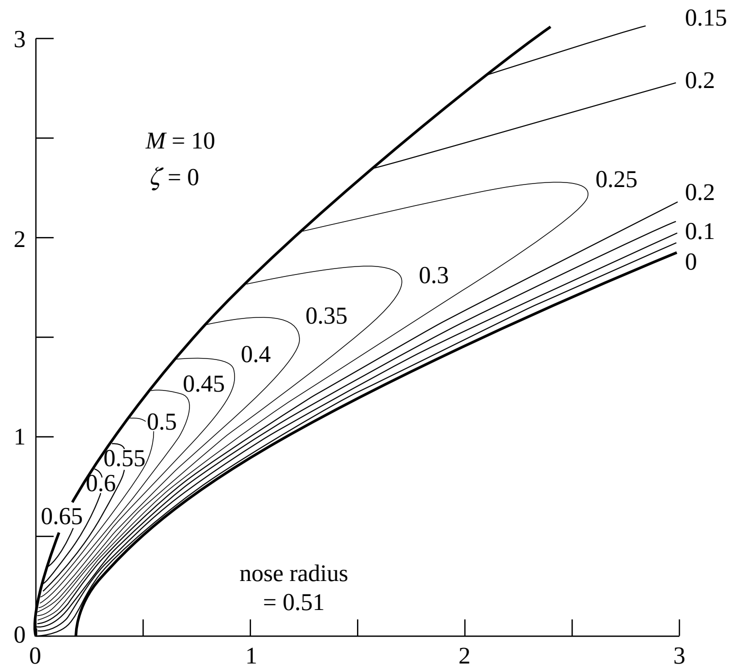

Figure 13. Lines of constant vorticity ζ in a flow between shock wave and round-nosed surface. (Values shown on each curve are of $r\zeta/U_\infty$, where r is the nose radius and U_∞ is the free-stream speed.)

become negligible, the round-nosed wedge generally has a higher heat transfer than one with a sharp nose, even though the surface slope and pressure are the same for both. However, the difference in the rate of heating is not large and reverses in sign if the wedge angle is large. The figure relates to the limit of infinite Mach number, and the difference also tends to reduce as the Mach number becomes smaller. This is because it is an effect of the entropy rise through the incident shock. In thin boundary-layer theory (upon which all our calculations are based), the entropy at the outside of the boundary layer is taken to be the same as that at the surface in inviscid flow. This is identical with the entropy at the stagnation point for the round-nosed wedge, whereas for a sharp-edged wedge, it is that immediately downstream of the plane shock.

Another way of viewing this difference is to suppose that the very thin entropy layer produced by the 'almost-sharp' wedge is swallowed by the boundary layer, whereas for the round-nosed wedge, the entropy layer lies outside the boundary layer. Figure 13 shows a typical vorticity distribution for inviscid flow near a round-nosed cylinder. Since the entropy gradient normal to the streamlines is proportional to the vorticity, clearly the bulk of the change in entropy takes place away from the surface. Moreover, it occurs over a region whose thickness is large compared with the nose radius.

In practice, it is not only difficult to predict which assumption to apply, but it is possible, of course, that neither truly represents the downstream condition. However, in what follows, we shall assume that the entropy at the edge of the boundary layer equals the stagnation point value. This generally avoids underestimating the heat

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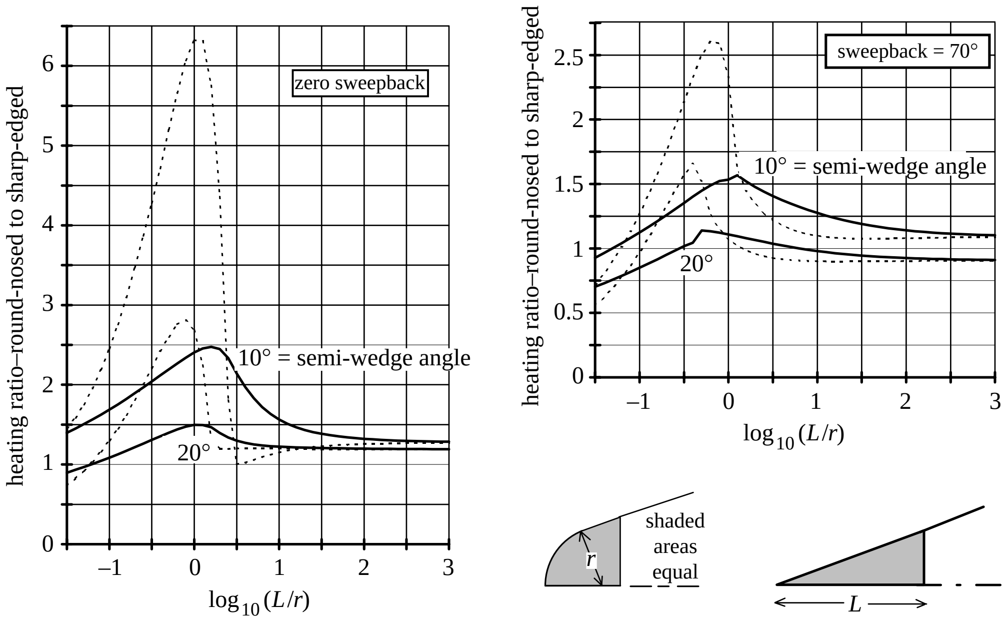

Figure 14. Heat transferred to a round-nosed wedge as a multiple of that to a sharp-edged wedge of the same included cross-section area A and same streamwise semi-wedge angle. (Dotted curve: ratio of rates of surface heating at downstream end of area A. Solid curves: ratio of total heat input to surface from nose to downstream end of A.)

transfer. More to the point, we do not have to guess how the entropy at the outside of the boundary layer changes from the stagnation point value, which doubtless applies over the nose, to the plane shock value that we assume to exist downstream.

(*b*) *Effect on heat transfer*

As in all of the other results we have quoted up to now, it is relevant for our purposes to compare the round-nosed wedge with a sharp-edged one having the same edge sweepback and wedge angle, and the same mass or cross-sectional area (A, say). If we assume that the wedge is cut-off flat at its downstream end, then $A = L^2 \tan\delta$ for the sharp-edged wedge. Although the expression for A is less simple for the rounded wedge and depends on its nose radius r, it follows that if it has the same cross-sectional area, then a value of L/r implies a position on both sharp and round-nosed wedges. The larger the value, the further either wedge extends downstream, relative to the radius r.

On this basis, we can compare the hypersonic heating rates derived from the simplified calculations used in drawing up figures 11 and 12, and the results are shown in figure 14. The solid lines indicate the ratio of the *total* heating rates. These are found by integrating along both round- and sharp-edged surfaces, from the nose to the downstream end. On the other hand, the dotted lines refer to the ratio of the *local* rates of heating at the downstream end. These local rates determine the radiation equilibrium temperature immediately downstream of the wedge, and the fourth root of their ratio is the ratio of these temperatures.

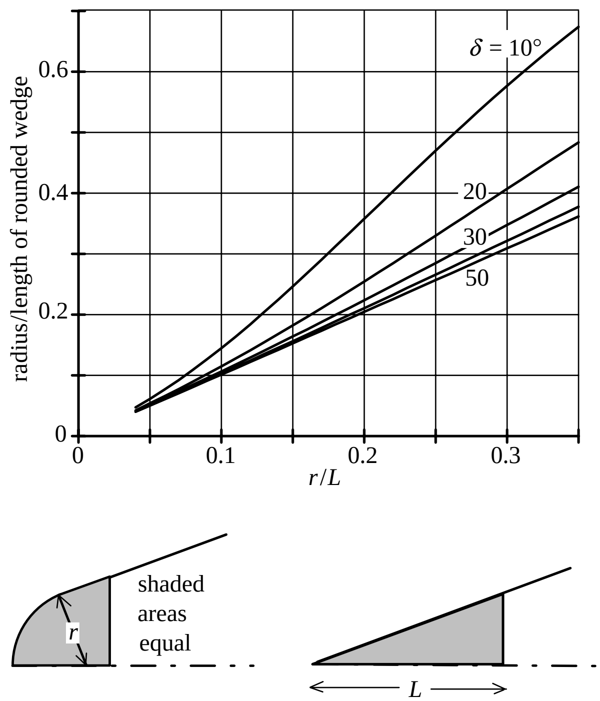

Figure 15. Radius-to-length ratio of rounded- and sharp-edged wedges of equal cross-sectional area as determined by the semi-wedge angle δ in the plane normal to the edge.

The value of either ratio is zero at the nose, since the sharp-edged wedge has an infinite rate of heat transfer at the nose. However, it exceeds unity for small L/r, implying that the heating on the rounded wedge becomes comparatively high at some position close to the stagnation point on its curved nose. It reaches a maximum at about $L/r = 1$ (corresponding roughly to an angle δ round the curved nose), and then falls away rather slowly to an asymptotic value, generally above unity. This corresponds to the higher heating of a round-nosed section far downstream, as shown in figure 12 (and discussed in the previous section).

The only exception to this general pattern is provided by the thicker swept wedge. The quoted *streamwise* semi-angles δ_s of 10° and 20° of figure 14 become† angles of 27° and 47° normal to the edge of a wing swept back through 70°. As figure 12 confirms, the wedge angle of 47° is large enough to lower the asymptotic heating rate on the round-edged wedge below that of the sharp wedge.

(*c*) *Effect on nose temperature*

We have already seen (in § 3) that, in the 'infinite conductivity' solution corresponding to $L/\Lambda \to 0$, the surface temperature is constant. Its value is such that the heat radiated from the wedge exactly balances that input. It follows from figure 14 that the provision of a nose radius cannot be effective in reducing the surface temperature if L/Λ is too small. (The only exception to this is likely to arise for semi-wedge angles δ exceeding 45° or so.) On the other hand, it is also clear from figure 14 that

† Upon assuming $\delta = \arctan(\tan\delta_s \sec\phi)$.

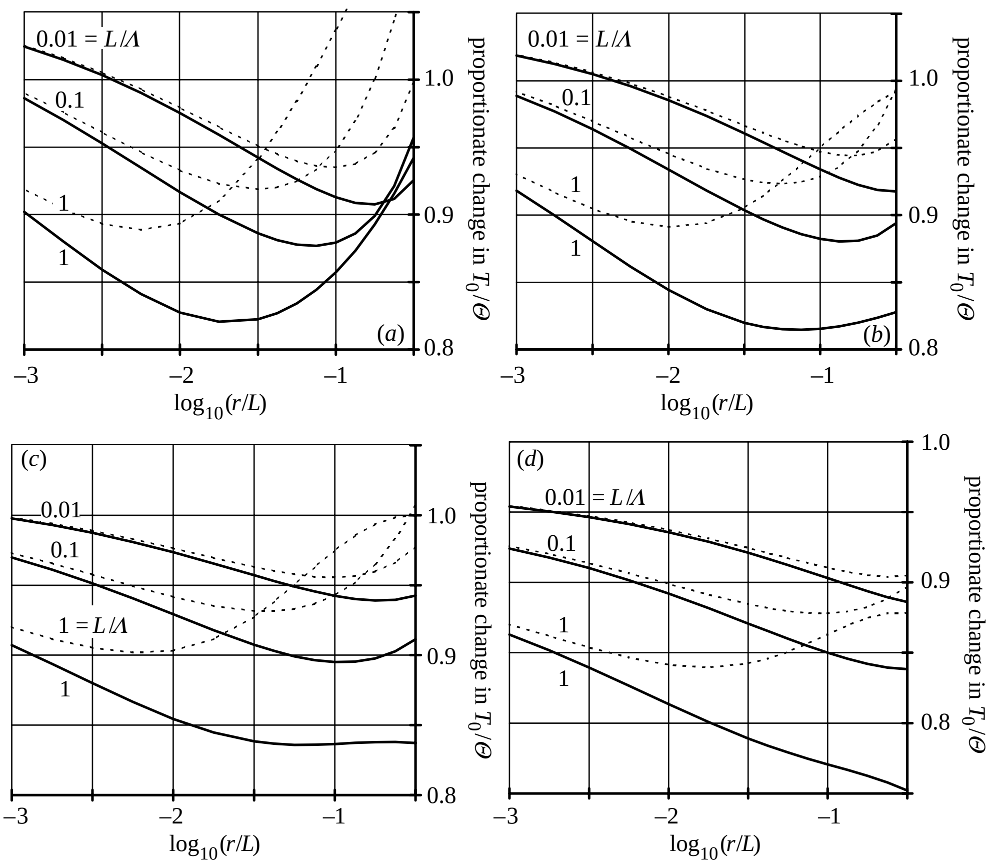

Figure 16. Change in T_0/Θ due to a circularly rounded nose on a wedge of streamwise semi-apex angle δ with its edge swept back by angle ϕ, keeping the same cross-section of conducting material. Unbroken lines, symmetrical heating; broken lines, one-sided heating. (*a*) $\delta = 10°$, $\phi = 0°$; (*b*) $\delta = 20°$, $\phi = 0°$. (*c*) $\delta_s = 10°$, $\phi = 70°$; (*d*) $\delta_s = 20°$, $\phi = 70°$.

any benefit is likely to disappear as r/L tends to 1 from below. Thus, there is likely to be a minimum temperature reached for some $r/L < 1$, even if the loss of accuracy in conducting-plate theory did not limit us to this range. Note that L refers here (as elsewhere) to the length of the sharp-edged wedge of the same cross-sectional area. As shown in figure 15, the length of the rounded wedge is less than L, and its radius-to-length ratio is, therefore, larger.

All of these trends are observable from figure 16*a*–*d*. By comparison with a sharp-nosed section, one with a rounded nose is effective in reducing the nose temperature only for $L/\Lambda > 0.01$. (As already anticipated, the only exception to this is provided by the extreme case of high sweep and large wedge angle shown in figure 16*d*.) There is a maximum reduction for some value of r/L in the interval (0.01,1). However, as r/L is decreased, it will be seen that the temperature ratio (of rounded to sharp-edged wedges) does not tend to unity, as one would have expected. This is because of the non-uniform convergence to the limit introduced by our assumptions (as discussed in § 5 *a*). However, no doubt it will do so in reality, even though we cannot model the process.

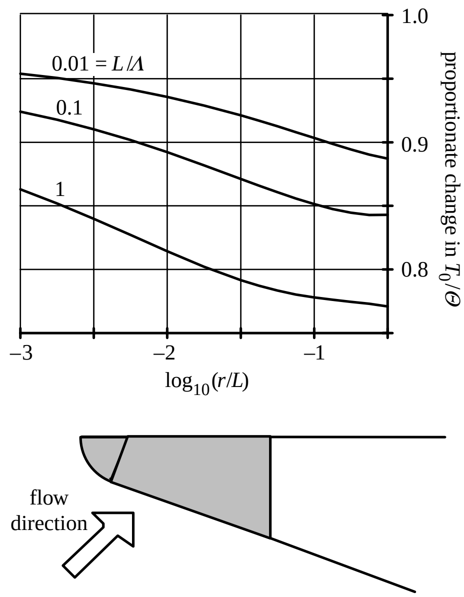

Figure 17. Proportionate change in T_0/Θ due to a circularly rounded nose on a flat-topped wedge of streamwise semi-apex angle $\delta_s = 10°$ with its edge swept back by angle $\phi = 70°$, keeping the same cross-sectional area of conducting material. (Heating on underside only.)

The solid lines on all parts of figure 16 relate to symmetrical heating of the top and bottom wedge surfaces. If the wedge is at such a high angle of attack that the heating is only to one side, then the reductions indicated by the broken lines apply. These are lower because there is still heating over the full curved region of the nose, and this becomes progressively more important as r/L tends to unity.

One-sided heating is almost certain to apply in hypersonic lifting re-entry, and the only way we can regain the advantage of the symmetrical arrangement is by halving the circular nose, as illustrated in figure 17. The curved surface here occupies less than a quadrant.† With this modification, it is seen by comparing figure 17 with 16*d* that, as could be expected, the reduction in T_0 is close to those on a symmetrical wedge with double the semi-wedge angle, δ.

It is found that the use of taper or the introduction of lighter materials remains an advantage on round-nosed sections, though generally rather less so than on sharp-edged wedges. Since the effect of rounding is most marked at larger values of L/Λ, there is in any event a greater advantage in using graphite as the conducting material, because its low density allows a larger L and its low conductivity reduces Λ. For instance, in the example provided by SLEEC22, the niobium nose backed by graphite yields a leading edge temperature of 1400 K (2520 °R) on a sharp-edged wedge of 10 kg m^{-1} mass. Since its L/Λ is 0.109, if it is given a 'half-rounded' nose with $r/L = 0.1$, its T_0 might be lowered by 15% to 1190 K (2140 °R), as indicated by figure 17 (which does not, however, apply precisely to this example). On the other hand, table 1 lists L/Λ as 0.46 for graphite and the nose temperature of a sharp-edged graphite wedge as 1505 K (2710 °R). Figure 17 suggests that its nose temperature can

† It is arguable that what we sketch as a sharp corner at the edge of the top surface should in fact be rounded, although with a much smaller radius than that of the lower quadrant.

be reduced by $17\frac{1}{2}\%$ with $r/L = 0.1$, down to 1240 K (2230 °R). Half the difference between the nose temperature of these two materials has now disappeared.

For the record, in its published form, Nonweiler (this issue, SLEEC22) quotes the mass of the leading edge conducting material as halved to $5\ \mathrm{kg\,m^{-1}}$, and that a sharp-edged niobium wedge backed by graphite would reach $T_0 = 1435$ K (2585 °R). It was assumed that rounding could provide a further 5% reduction in T_0 to 1363 K (2453 °R). Figure 17 confirms that this is clearly well within the bounds of possibility.

6. Designing the wing apex

In considering the heating of a swept leading edge, we have so far treated conduction as a two-dimensional process. If the wing apex is pointed in plan view, then this assumption must break down close to that point. However, suppose that the leading edge of the wing, looked at in plan view, is curved so gradually that its radius of curvature is large compared with the conduction length. Then there is no reason why conductivity should not remain a locally two-dimensional effect, up to and including the now-rounded apex. However, the provision of conducting material will need to vary spanwise, along the curved leading edge, as the local sweepback changes. Certainly, the wing apex itself will always present the most difficult condition to deal with.

This is perhaps most easily illustrated by again using the re-entry vehicle SLEEC22 (Nonweiler, this issue) as an example. At its (rounded) apex, the heating intensity is held to within $H = 50\ \mathrm{kW\,m^{-3/2}}$, and the now streamwise semi-wedge angle is 0.15 rad. If niobium is used, the conducting length Λ is now only 7.5 cm (3 in) and the reference temperature is 1175 K. No heavy mass is needed to provide a value of L/Λ of unity or more, and a sharp wedge could therefore have a nose temperature of 2000 K (3600 °R). Figure 16a, b suggests that a 'half-rounded' nose with $r/L = 0.1$ might be a little above 1670 K (3000 °R). More accurate calculations, including real-gas effects and use of a graphite backing to the niobium, put this figure nearer 1750 K (3100 °R).

However, the problem is not so much the value of T_0 at the apex as the extensive region of the surface that is unacceptably hot downstream of the conducting region. The region being protected needs to be lengthened well beyond $L = \Lambda$, although we do not need to install conducting material for this purpose. However, there is merit in doing so, because we might use a redundantly large value of (say) $L = 3\Lambda$ at hypersonic speeds, merely to be able also to cool the apex at lower Mach numbers (where Λ is larger, because H is lower). The same considerations apply to a swept-wing leading edge, but apex temperatures are higher and, generally, remain a problem down to quite low Mach numbers.

In extreme conditions, it might be necessary to cool the apex by circulating a coolant. In this condition, the use of conducting materials can continue to play an important role. However, it would take us too far afield to follow this suggestion any further here, though it might find application in scramjet design and the RBCC (see, for example, figure 3, p. 2323).

My grateful thanks are due to Leo Townend of APECS Ltd. On many occasions and over many years, his unfailing, though not uncritical, support of this line of research has rekindled my enthusiasm, even when its embers have become rather lifeless.

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Appendix A. The application of conducting-plate theory

We consider a thin two-dimensional body and we describe an x-axis of an orthogonal Cartesian coordinate system within it. We suppose that the y-axis meets the top and bottom surface of the body at $y = y_{\rm t}(x) > y_{\rm b}(x)$ for all $x \in (0, L)$, and we write the body thickness as $t(x) = y_{\rm t}(x) - y_{\rm b}(x)$. Usually, we shall take $y_{\rm t}(0) = y_{\rm b}(0) = t(0) = 0$, and we assume that $t(x) \ll L$ for all $0 \leqslant x \leqslant L$. The surface slopes are taken as

$$\beta_{\rm t}(x) = \arctan({\rm d}y_{\rm t}/{\rm d}x),$$

and, similarly,

$$\beta_{\rm b}(x) = \arctan({\rm d}y_{\rm b}/{\rm d}x).$$

The lengths of the body surface intercepted between two ordinates at distance x and $x + {\rm d}x$ from the origin, where ${\rm d}x$ is infinitesimal, are $\sec\beta_{\rm t}\,{\rm d}x$ and $\sec\beta_{\rm b}\,{\rm d}x$.

We suppose next that in the plane $x =$ const., the body is subjected to steady rates of heat input $Q_{\rm t}(x)$, $Q_{\rm b}(x)$, and loses heat by radiation at the steady rate $R_{\rm t}(x)$, $R_{\rm b}(x)$, per unit surface area of its top and bottom surfaces, respectively. Then, if $\eta(x)$ is the conduction of heat in the x-direction within the body across an ordinate at distance x from the nose, the equation of steady heat conduction states that, as ${\rm d}x \to 0$,

$$\eta(x + {\rm d}x) - \eta(x) = {\rm d}\eta(x) = [Q_{\rm t}(x) - R_{\rm t}(x)]\sec\beta_{\rm t}\,{\rm d}x + [Q_{\rm b}(x) - R_{\rm b}(x)]\sec\beta_{\rm b}\,{\rm d}x. \tag{A 1}$$

We now assume that the distribution of material within the body is such that we can (approximately) treat its temperature T as being independent of y. Then,

$$R_{\rm t}(x) = \varepsilon_{\rm t}\sigma T^4(x),$$

and, similarly,

$$R_{\rm b}(x) = \varepsilon_{\rm b}\sigma T^4(x),$$

where $\varepsilon_{\rm t}$ and $\varepsilon_{\rm b}$ are surface emissivities, and σ is the Stefan–Boltzmann constant. Further, the heat conduction is $\eta(x) = -k(x)t(x)\,{\rm d}T/{\rm d}x$, where $k(x)$ is the integral mean of the material conductivity with respect to y from $y_{\rm b}(x)$ to $y_{\rm t}(x)$. It will be realized that our assumptions impose constraints on the variation of conductivity in the y-direction. In particular, the existence of insulation or a gap (i.e. a region of small or zero k) could result in a discrete, and possibly large, change in temperature from one side to the other.

The basic equation of conducting-plate theory is, therefore:

$${\rm d}[k(x)t(x)\,{\rm d}T/{\rm d}x]/{\rm d}x = [\varepsilon_{\rm t}\sigma T^4(x) - Q_{\rm t}(x)]\sec\beta_{\rm t} + [\varepsilon_{\rm b}\sigma T^4(x) - Q_{\rm b}(x)]\sec\beta_{\rm b}. \tag{A 2}$$

We solve this equation subject to the boundary conditions $\eta(0) = \eta(L) = 0$. The former of these conditions ensures that the temperature remains bounded at the nose, and the latter imposes the condition that there is no transfer of heat downstream.

For instance, as applied to the heating of a symmetric homogeneous wedge of semi-angle δ by a boundary layer, we take

$$Q_{\rm t}(x) = Q_{\rm b}(x) = \tfrac{1}{2}H/\sqrt{r},$$

where $r = x \sec \delta$ is the radial distance along the surface. It is in fact easier to replace x by r in (A 2), which then becomes

$$2k \tan \delta \, \frac{\mathrm{d}(r \, \mathrm{d}T/\mathrm{d}r)}{\mathrm{d}r} = \Sigma \sigma T^4 - \frac{H}{\sqrt{r}}, \tag{A 3}$$

where $\Sigma = \varepsilon_{\mathrm{t}} + \varepsilon_{\mathrm{b}}$. However, since β is here everywhere small (and equal to $\pm\delta$) because of our assumption that the surface is thin, we would be justified in recasting this equation as

$$2k\delta \, \frac{\mathrm{d}(x \, \mathrm{d}T/\mathrm{d}x)}{\mathrm{d}x} = \Sigma \sigma T^4 - \frac{H}{\sqrt{x}}. \tag{A 4}$$

In this form, we can also relax the requirement of symmetry and assume that $Q_{\mathrm{t}}(x) + Q_{\mathrm{b}}(x)$ is equal to $H/\sqrt{x}$. This is the equation governing the 'reference solution' (3.1) of the main text. Using the definitions of the characteristic temperature Θ and conducting length Λ given in (3.2) of the main text, we find, on placing $z^2 = x/\Lambda$ and $\tau = T/\Theta$, that (A 4) becomes, quite simply,

$$\frac{\mathrm{d}(z \, \mathrm{d}\tau/\mathrm{d}z)}{\mathrm{d}z} = 2(z\tau^4 - 1). \tag{A 5}$$

The condition that $\eta(0) = 0$ is satisfied here by taking $\tau'(0) = -2$, and the condition that $\eta(L)$ is zero, becomes $\tau'(z) = 0$ at $z = \sqrt{(L/\Lambda)}$. This is probably most easily solved by the shooting method, guessing an initial τ at $z = 0$, and iterating to satisfy the downstream condition on τ'. The author prefers to express the second-order equation as a system of two first-order equations in (non-dimensional) variables for T and η (rather than T and $\mathrm{d}T/\mathrm{d}z$). If there is a discontinuous change in $k(x)$, then η will be continuous, but not $\mathrm{d}T/\mathrm{d}z$.

Other problems are formulated similarly as special cases of (A2). If, as in the example of a thin wedge, the surface slope is everywhere small, we can omit the factors of $\cos\beta$. Either in that event or when the body is symmetric so that $\beta_{\mathrm{t}} = \beta_{\mathrm{b}}$, we can group the emissivities and rates of heating together, as

$$\Sigma = \varepsilon_{\mathrm{t}} + \varepsilon_{\mathrm{b}} \quad \text{and} \quad Q = Q_{\mathrm{t}}(x) + Q_{\mathrm{b}}(x).$$

For instance, in the (symmetric) heating of a circularly rounded nose, (A 2) becomes

$$2\frac{\mathrm{d}(k \, \mathrm{d}T/\mathrm{d}\theta)}{\mathrm{d}\theta} = r_0(\Sigma \sigma T^4 - Q), \tag{A 6}$$

for $0 \leqslant \theta \leqslant \pi/2 - \delta$, where θ is the angular coordinate round the nose. At $\theta = 0$, the boundary condition that $\eta = 2k \, \mathrm{d}T/\mathrm{d}\theta$ is 0 simply ensures that $\mathrm{d}T/\mathrm{d}\theta = 0$, as is consistent with symmetry. On the surface of the wedge downstream, and with the assumption that δ is small, it is convenient to place $t(x) = 2y_{\mathrm{s}} + 2(z^2 - z_{\mathrm{s}}^2)\Lambda\delta$ where $z_{\mathrm{s}}^2 = x_{\mathrm{s}}/\Lambda$ and $x_{\mathrm{s}} = r_0(1 - \sin\delta)$ and $y_{\mathrm{s}} = r_0 \cos\delta$ are the coordinates of the shoulder. Treating δ as small, we can place $x_{\mathrm{s}}, y_{\mathrm{s}} \approx r_0$. Then, we continue the solution as

$$\frac{\mathrm{d}\{[r_0/(\Lambda\delta) + z^2 - z_{\mathrm{s}}^2]z^{-1} \, \mathrm{d}\tau/\mathrm{d}z\}}{\mathrm{d}z} = 2z(\tau^4 - \Lambda^{1/2}Q/H), \tag{A 7}$$

for $r_0 \leqslant \Lambda z^2 \leqslant L$, where $\Lambda^{1/2}Q \to H/z$ as $z \to \infty$. The boundary conditions to be met are that η is continuous at the shoulder and zero at $x = L$. In this way, the solution merges smoothly with that of a thin pointed wedge as $r_0 \to 0$.

It is arguable whether conducting-plate theory is correctly applicable to a blunt or rounded shape (Nonweiler 1952, 1956). The justification rests largely on a restriction to relatively small values of nose radius, which, as we see from (A 6), implies that the change in temperature over a rounded nose is relatively small, although it needs to be noted that Q increases in proportion to $1/\sqrt{r_0}$. However, as illustrated in figure 11*b*, it usually happens that Q drops over the curved region of the nose roughly in proportion to x (which varies as $\cos\theta$). This in turn means that the variation of T, as determined by a solution of (A 6), also has a dominant component varying as $-x$, so that (fortuitously perhaps) T may be quite closely a function of x, as the theory assumes.

Notation

A	cross-sectional area of conducting material
E_n	radiation equilibrium temperature at a normal distance n from the nose
E_r	radiation equilibrium temperature at a normal distance r from the nose
H	heating intensity, assumed constant, being the total heat input to both surfaces of the wedge within the normal distance n from the nose, divided by $2\sqrt{n}$
H_s	heating intensity, assumed constant, being the total heat input to both surfaces of the wedge within the streamwise distance s from the nose, divided by $2\sqrt{s}$
L	length of wedge in the x-direction normal to its edge
Q	rate of heat transfer from the boundary layer
T	absolute temperature
T_0	absolute temperature at the nose
V_c	(nominal) circling velocity
V_∞	free-stream speed
k	material conductivity
m	mass of conducting material per unit length along edge
p_e	surface pressure (at edge of boundary layer)
r	$= n$, radial normal distance from nose, *and* nose radius (from § 5)
s	distance from edge in streamwise direction
s_g	specific gravity of the conducting material
x	distance along the x-axis described within the wedge and normal to its edge
Λ	conducting length, defined by (3.2)
Θ	$= E_\Lambda$, reference temperature, defined by (3.2)
Σ	$= \varepsilon_b + \varepsilon_t$, the sum of the surface emissivities
δ	semi-wedge angle of wedge in plane normal to edge
δ_s	semi-wedge angle of wedge in streamwise direction
ε	surface emissivity
ϕ	local sweepback of edge of wedge
ρ	material density

σ the Stefan–Boltzmann constant, 5.6697×10^{-8} W m^{-2} K^{-4} (1.7121×10^{-9} Btu h^{-1} ft^{-2} $^{\circ}$R^{-4}).

Additionally, the subscripts 'b' and 't' applied to Q, ε and p denote values on the bottom and top wedge surfaces, respectively.

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